18. (20 points) If the vertex factor for the decay of a vector boson $X$ into two spin-$1/2$ fermions $f_1$ and $f_2$ is

$$-ig_X\gamma^\mu\frac{1}{2}(g_V - g_A\gamma^5),$$

then show that

$$\Gamma(X \to f_1f_2) = \frac{g_X^2}{48\pi}(g_V^2 + g_A^2)M_X,$$

where $M_X$ is the mass of the boson and the masses of the fermions have been neglected. This is Halzen and Martin Exercise 13.2, and you can use the hints given there. This formula is very useful since it can be used for both $W^\pm$ and $Z^0$.

19. (10 points) Assuming the Standard Model coupling and using the result of Problem 18, show that

$$\Gamma(Z^0 \to \nu_e\bar{\nu}_e) = \frac{g^2}{96\pi\cos^2\theta_W}M_Z.$$

Calculate the numerical value for this partial width, using $\sin^2\theta_W = 0.232$ and $M_Z = 91.2$ GeV.

20. (10 points) Calculate the partial widths for the three decay modes $Z^0 \to e^+e^-, u\bar{u}$, and $d\bar{d}$, and use these to calculate the Standard Model value for the total width of the $Z^0$. Do not forget color. Compare the calculated value with the measured value and give an explanation for any difference.

21. (10 points) Using the result of Problem 18, calculate the partial widths for $W^+ \to e^+\nu_e, u\bar{u}$, and $\bar{u}u$ and thus the total Standard Model value for the total width of the $W^+$. Use $M_W = 80.4$ GeV. Do not forget the Cabibbo angle factors for the quark-antiquark modes. Compare with the measured value.