11. (20 points) In class we discussed $e^{-}\pi^{+}\rightarrow e^{-}\pi^{+}$ (π has spin = 0) for a "point-like" pion, for which the spin-averaged matrix-element squared is given by

$$\langle M \rangle^2 = \frac{e^4}{q^4} L_{\mu\nu}^{\mu\nu},$$

where

$$L_{\mu\nu}^{\mu\nu} = 2 \left( k^\mu k^\nu + k'^\mu k'^\nu + \frac{q^2}{2} g^{\mu\nu} \right)$$

and

$$T_{\mu\nu} = (p' + p)_\mu (p' + p)_\nu.$$

$k$ and $k'$ are the initial and final four-momenta of the electron, $p$ and $p'$ are the initial and final four-momenta of the pion, and $q$ is the four-momentum of the exchanged photon. Show that in the laboratory frame (the center-of-mass system of the initial pion)

$$\left. \frac{d\sigma}{d\Omega_{\text{lab}}} \right|_{\text{lab}} = \frac{\alpha^2}{4 E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{\cos^2 \frac{\theta}{2}} \frac{E^2}{2},$$

where $E$ and $E'$ are the initial and final energies of the electron and $\theta$ is the angle between the initial and final electron directions. Ignore the mass of the electron.

12. (10 points) In inelastic $e p$ scattering it is common to replace $\nu$ and $q^2$ by the dimensionless variables

$$x = \frac{-q^2}{2 p \cdot q} = \frac{-q^2}{2 M \nu}, \quad y = \frac{p \cdot q}{p \cdot k} = \frac{\nu}{E (\text{lab})},$$

where the four-momenta are as given in lecture. Show that the allowed kinematic region for $e p \rightarrow e X$ is $0 \leq x \leq 1$ and $0 \leq y \leq 1$. (This is Halzen and Martin Exercise 8.11)

13. (20 points) Read the paper by M. Breidenbach et al., "Observed Behavior of Highly Inelastic Electron-Proton Scattering," Phys. Rev. Lett. 23, 935 (1969). You can find this reprinted in Cahn and Goldhaber, The Experimental Foundations of Particle Physics, pp. 245-249, and you will also receive a copy. The preceding paper in Cahn and Goldhaber,

Describe the evidence in this paper for scaling behavior.