Fragmentation @ LEP

Klaus Hamacher
Bergische Universität Wuppertal & DELPHI Collaboration

Outline

- Energy dependence of the charged hadron multiplicity and of the momentum spectra (ADLO)
- Gluon fragmentation function from 3 jet events (O)
- Colour coherence in 3 jet events (D)
- The multiplicity in 3 jet events, of gluon-jets, and gluon to quark ratios (D)
- Summary
The Charged Multiplicity

Multiplicity increase in $e^+e^- \rightarrow q\bar{q}$ due to coherent gluon bremsstrahlung off quarks

$$\left| \begin{array}{c} 0000 \end{array} \right| \propto C_F \cdot \alpha_s$$

Consistent description of the energy dependence of the multiplicity by:

- Fragmentation models
- MLLA (+ LPHD: $\#_{\text{hadrons}} \propto \#_{\text{gluons}}$)

$$\langle N_{ch} \rangle = K_0 \cdot \alpha_s(E_{cm})^{C_1} \cdot e^{C_2 \sqrt{\alpha_s(E_{cm})}}$$

- higher order (3NLO) predictions.

Dependence on flavour composition small but visible
Momentum Spectra – Scaling Violations

Scaling violations clearly observed in comparison of \( \sim 200\text{GeV} \) and \( Z \) data.

Gluon radiation →
- depletion of FF at high \( x \)
- multiplicity increase at small \( x \)

Data well described by Monte Carlo’s

Scaling violations underestimated by FF parameterisations (GDLAP based)
Momentum Spectra – Small $x$

Colour coherence limits gluon emission at small $x$ / large

$$\xi = -\ln x = -\ln \frac{2E_h}{E_{cm}}$$

Predicted by MLLA “limiting spectrum”

Coherence $\rightarrow$ change of peak position

$$\xi^*(E) \sim \sqrt{\log E}$$

slower than expected from phase space

$$\xi^*(E) \sim \log E$$

Klaus Hamacher, Fragmentation @ LEP: Gluon Fragmentation Function
Comparing Gluons and Quarks in $e^+e^- 3$ Jet Events

- assign partons ↔ jets at tree level
- identify quarks using E-ordering and displaced vertices (heavy q’s)
- determine parton kinematics from event topology
- unfold light-, b-quark, g-contribution by inverting purity matrix
- dynamical studies require evolution scales for jets → transverse momentum like scales:
  \[ \kappa = Q = E_{jet} \cdot \sin \frac{\theta_{ij}}{2} \left( \equiv \sqrt{s} \right) \]

- assignment of particles to jets requires jet algorithms → ambiguities

These can be avoided/minimised by
- analysing only 3-jet event multiplicity (DO*)
- use fully symmetric situation needs boost algorithm (O)
- use recoiling gluons (O)
- analysing fast hadrons only (Fragmentation Functions) (DO*)
Gluon and Quark Fragm. Funct. in 3 Jet Events

- OPAL: new Frag. Func. analysis for incl., udsc, b and gluon
  - inclusive events → quark FF’s
  - jets of 3 jet events → quark and gluon FF’s

- $Z$ and high $E$ data combined, cover $E_{CM}$ equivalent from:
  - quarks: 10...210 GeV
  - gluons: 10...100 GeV

- in 3 jet events use scale suggested by angular ordering
  $$\kappa = Q = E_{jet} \cdot \sin \frac{\theta_{ij}}{2}$$

- inclusive and 3-jet results agree well for quarks!

- available FF parameterisations reasonably describe quark data.
Gluon and Quark Fraggm. Funct. in 3 Jet Events

- New gluon results in agreement with previous results:
  - OPAL’s “inclusive” and “boosted” analysis
  - similar DELPHI 3 jet analysis

- Extends available kinematic range

- Parameterisations describe gluon data less well

- Energy slope at high $x$ stronger than expected (both OPAL and DELPHI)
Check for biases of 3 jet analysis

- Compare 3 jet results to inclusive $q\bar{q}$ or $gg$ events in MC
  - agreement OK for quarks
  - sizable difference for gluons at high $x$ small $E$

OPAL

Pythia6.1  Herwig 6.2
three-jet ev.  
inclusive ev.  
\( \chi \)

\[
\frac{1}{N_{jet}} \frac{dN_{ch}}{dx_E} \quad \text{scale [GeV]}
\]

- WHY? wrong scales, mass effects or experimental?
- OPAL’s “boost” analysis sees similar bias
- It's an experimental effect!
  Data not unfolded down to parton level
  all analyses assume $E_{jet} = E_{parton}$

But: full unfolding required
  $\rightarrow$ for exact definition of $x = E_{hadron}/E_{parton}$

Common problem of all 3 jet analyses!

- In practice only affects high $x$ gluon FF
- Requires improvement of exptl. procedures!
  Still feasible for LEP experiments?
Coherent Particle Production at Large Angles

- Soft gluons at large angles:
  Large wave-length $\rightarrow$ small resolution $\rightarrow$ coherent emission
  Effective colour-charge depends on event topology

- Compare gluon radiation $\perp$ to $q\bar{q}g$ plane $\leftrightarrow \perp$ to $q\bar{q}$ axis.

\[
\frac{N_{\perp}^{q\bar{q}g}}{N_{\perp}^{qq}} = \frac{C_A}{C_F} \cdot r_t = \frac{C_A}{C_F} \cdot \frac{1}{4} \left[ \frac{1}{N_C^2} \right] \quad (i \cdot j) = 2 \sin^2 \frac{\vartheta_{ij}}{2}
\]

Ratio directly proportional to $C_A/C_F$ in LO. No corrections!

Destructive gluon-interference term $\propto 1/N_C^2$

- Experimentally identify partons with $k_t$-jets (at fixed $y_{cut}$):
  defines 2 and 3-jet events, excludes $\geq 4$-jet events $\leftrightarrow$ LO

  Compare multiplicity $\perp$ to 3-jet plane to the one $\perp$ 2-jet axis

Khoze, Ochs, Lupia
The Destructive Interference Term $\propto 1/N_C^2$

- Solid line: complete $r_t$, dashed line: without $1/N_C^2$-term

- Data only described with interference
  Fit amplitude $k$ of $1/N_C^2$-term:

  - general topologies:
    $$k = 1.39 \pm 0.05_{\text{stat.}} \pm 0.28_{\text{sys.}}$$
  - symmetric topologies:
    $$k = 1.30 \pm 0.06_{\text{stat.}} \pm 0.33_{\text{sys.}}$$

  Syst. error from variation of $y_{\text{cut}}$, $\theta_{\text{cone}}$ and cluster algorithms

- Compatible with $k = 1$
  $k = 0$ excluded with $> 95\% CL$.

New clear observation of colour coherence!
The Connection to $C_A/C_F$

- Expect homogenous linear relation

$$\frac{N_3}{N_2} = r_t \cdot \frac{C_A}{C_F} = r_t \cdot \text{slope}$$

- Data incl. stat. and syst. errors.
- Measured slope from fit:

  general topologies:
  $$2.182 \pm 0.009_{\text{(stat.)}} \pm 0.055_{\text{(sys.)}}$$

  symm. topologies:
  $$2.205 \pm 0.006_{\text{(stat.)}} \pm 0.073_{\text{(sys.)}}$$

Amazingly good agreement with expectation

slope $= C_A/C_F = 2.25$
Momentum-Distribution as Function of $r_t$

- Use $r_t$ as single scale
- Momentum distributions scale for $p \lesssim 1\text{GeV}$
  
  Right: renormalised to average/theo. expectation

- Pert. expectation fulfilled for very low energy hadrons $\rightarrow$
  
  Local Parton Hadron Duality

Klaus Hamacher, Fragmentation @ LEP: Coherent Soft Particles
ISMD 2004, Sonoma Univ., 28.7 – 1.8 2004 12
Topology Dependence of 3 Jet Event Multiplicity

Prediction accounts for coherence effects by choice of scales

\[ N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Le}) = N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu}) + \frac{1}{2} N_{gg}(\kappa_{Le}) \]  \hspace{1cm} (A)

\[ N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Lu}) = N_{q\bar{q}}(L, \kappa_{Lu}) + \frac{1}{2} N_{gg}(\kappa_{Lu}) \]  \hspace{1cm} (B)

with \[ L = \ln \left( \frac{s}{\Lambda^2} \right), \quad L_{q\bar{q}} = \ln \left( \frac{s_{q\bar{q}}}{\Lambda^2} \right), \]

\[ \kappa_{Lu} = \ln \left( \frac{s_{gg}s_{qg}}{s_{q\bar{q}}\Lambda^2} \right), \quad \kappa_{Le} = \ln \left( \frac{s_{gg}s_{qg}}{s_{q\bar{q}}\Lambda^2} \right) \]

Division of multiplicity in \( q\bar{q} \) and gluon part is arbitrary

\[ \rightarrow \text{differing definitions of gluon multiplicity} \leftrightarrow \text{differing scales} \]

The phase space of the \( q\bar{q} \)-pair is restricted by the gluon jet \( \rightarrow \text{requires correction} \)
Topological Dependence of 3 Jet Event Multiplicity

- In the Dipole Model, energy slopes of $gg$ and $q\bar{q}$ systems are related by:

$$\frac{dN_{gg}(L')}{{dL'}} \bigg|_{L'=L+c_g-c_q} = \frac{C_A}{C_F} \left( 1 - \frac{\alpha_0 c_r}{L} \right) \frac{d}{dL} N_{q\bar{q}}(L)$$

- $N_{q\bar{q}}(E_{cm})$ measured by various $e^+e^-$-experiments
- Solution leaves constant of integration free
  - To be determined from a single measurement of $N_{gg}$
  - Take CLEO-data from $\chi_b'(J=2) \rightarrow gg$ decay at $E_{cm} = 9.9132$GeV

Analysis:

- Select 3 jet events without cut on $y_{cut}$ (AoD, Cambridge, Durham, PHYJET)
- Compare general and symmetric topologies
- Compare $udscb$ and $udsc$ events $\rightarrow$ constant offset $N_0 \sim 0.6$ due to $b$-events
- Compare solutions Eden (A) and (B)
- Leave $N_0$ free $\rightarrow$ use slope for measurement of $C_A/C_F$
The 3 Jet Multiplicity

- Compare $udsc$ (○) and $udscb$ (●) data
- Eden A ———
  - Very good agreement for symmetric and general topologies
- Eden B - - -
  - Multiplicity overestimated by $\sim 0.6$
  - Slope too high
  - $\chi^2$ unacceptable in global fit
- Solution Eden B is incompatible with DELPHI data!
- Further DELPHI analysis bases only on Eden A
- OPAL used Eden B (sym. events only)
Is $N_0$ Constant?

The Connection to $\delta_{bl}$

- No topology dependence of the multiplicity difference $\delta_{bl}$ of $udscb$ and $uds(c)$ events is observed.

- Consistent with:
  - previous measurements of $\delta_{bl}$
  - QCD expectation.

- Probably a new precise measurement of $\delta_{bl}$ at reduced C.M.S. energy.

Klaus Hamacher, Fragmentation @ LEP: Multiplicity of 3 Jet Events

ISMD 2004, Sonoma Univ., 28.7 – 1.8 2004  16
Result for $C_A/C_F$

- We want $C_A/C_F$ from slope w.r.t. angles: $\rightarrow N_0$ varied freely!
- Experimentally advantageous:
  $\rightarrow$ insensitive to normalisation
  $\rightarrow$ no b-tagging systematics

\[
\frac{C_A}{C_F} = 2.261 \pm 0.014_{\text{stat.}} \pm 0.036_{\text{exp.}}
\]
\[
\pm 0.052_{\text{theo.}} \pm 0.041_{\text{clus.}}
\]
- Most precise measurement ($\sim 3\%$)
- Fixes QCD group structure to SU(3) together with measurements of $\beta$-function, 4 jet $\angle$-distributions
Energy dependence of the Gluon Multiplicity $N_{gg}$

- Determine Gluon contribution:

$$N_{gg}(\kappa_{Le}) = 2 \cdot (N_{q\bar{q}}(\theta_1) - N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu}) - N_0)$$

- Agreement between:
  - general & symmetric topologies
  - experiments (except similar OPAL anal.)
  - data and Eden prediction

- Gluon measurements start to compete with quark data

- $N_{gg}$: E-slope $\sim$ twice that of $N_{q\bar{q}}$
  $\rightarrow$ illustrates colour factor ratio
Ratio of the Multiplicities $r$ and the Slopes $r^{(1)}$

- $r$:
  - Large differences between LO, NLO, 3NLO, numeric predictions
  - Data only described by Eden (using experimental input)
  - Non-pert. effects are important!

- $r^{(1)}$:
  - Direct measurement of the linear slopes
  - Difference to $C_A/C_F$ smaller
  - 3NLO & Eden pred. similar in agreement with data
  - Non-pert. effects less important
  - Precision of data does not allow to measure 2’nd derivative $r^{(2)}$
Comparing Ratios $r$, $r^{(1)}$, \ldots $r^{(2)}$?

- Fit of 3NLO-prediction of $N_{q\bar{q}}$ and $N_{gg}$ to data

- Good description of data with $\Lambda_q \neq \Lambda_g$

- $r$, $r^{(1)}$ and $r^{(2)}$ can be calculated

- But:
  Determination of $r^{(2)}$
  based on used parametrisation
  \(\rightarrow\) correlation with the multiplicity measurement

Klaus Hamacher, Fragmentation @ LEP: Multiplicity of 3 Jet Events
Summary

- Multiplicity and fragmentation functions of quarks & gluons were measured over a wide energy range @ LEP
  - observed scaling violations (q & g), coherence effects $\xi^*$
  - gluon measurements to be improved (?)

- New measurement of colour coherence from soft particles in 3 jet events
  - destructive interference term observed
  - amazing agreement with LO prediction $\leftrightarrow$ LPHD

- Consistent measurements of the energy dependence of the gluon multiplicity
  - gluon slope $\sim$ twice quark slope
  - new precise measurement of $C_A/C_F$
  - QCD gauge group is fixed by measurement of hadronic properties: gluon/quark multiplicity, $\beta$-function
Gluon and Quark Fragm. Funct. in 3 Jet Events

Observed discrepancy is due to incomplete experimental unfolding!

- Jet energy reconstructed from observed hadrons.
- Assume \( E_{\text{jet}} = E_{\text{parton}} \)
- But smearing due to hadronisation \( \angle \)-resolution roughly 3\(^\circ\). \( \rightarrow \) smearing of \( E_{\text{parton}} \propto 1/E \)
  \( \rightarrow \) smearing of FF’s; \( \propto \) FF fall-off
  \( \Rightarrow \) strong overestimate of gluon FF at small \( E \) and high \( x \)
- The principle problem is there for all 3 jet analyses!
- Needs improvement of exptl. procedures!
  Still possible?

Klaus Hamacher, Fragmentation @ LEP: Additional Plots
Detailled Comparison to Opal

N_{\text{ch.}} e^+ e^-

- OPAL (uds)
- DELPHI (udsc)
- Eden A
- Param.
- without hadr. corr

\( \sqrt{s} \) [GeV]

- DELPHI 3Jet
- OPAL 3Jet
- OPAL Hemis.
- OPAL boosted
- CLEO
- MLLA-prediction

\( \theta_1[^{\circ}] \)

- qq events
  - ARGUS
  - Corrected data (c\&b quark contributions removed)
  - JADE
  - TASSO
  - HRS
  - AMY
  - LEP-averaged
  - DELPHI Param.

- 3NLO fit
- Jetset 7.4 uds cb events
- Jetset 7.4 uds events

Klaus Hamacher, Fragmentation @ LEP: Additional Plots
The $C_A/C_F$ Fit to the 3 Jet Multiplicity

Klaus Hamacher, Fragmentation @ LEP: Additional Plots