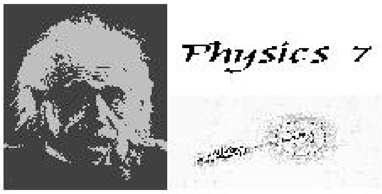

Chapter 4

Galileo and Newton

4.1 Introduction

The discoveries of Kepler, and the paradigm of the solar system of Copernicus provided a very solid framework for the works of Newton and Galileo. The resulting theories changed the way we do science to this day and some of their ideas have withstood the passing of time with little change.

4.2 Galileo Galilei

Only rarely humankind is fortunate to witness the birth and flourishing of a mind as keen and fertile as Galileo's. To him we owe our current notions about motion and the concepts of velocity and acceleration. He was the first to use the telescope as an astronomical tool. Galileo was also creative in devising practical machines: he invented the first accurate clock, an efficient water pump, a precision compass and a thermometer. These achievements distinguish him as the preeminent scietist of his time.

Galileo's research in the exact sciences banished the last vestiges of Aristotelian "science" and replaced it with a framework within which the whole of physics would be constructed. These changes were not achieved without pain: Galileo was judged and condemned by the Inquisition and died while under house arrest after being forced to recant his Copernican beliefs.

Underlying all the discoveries made by Galileo there was a modern philosophy of science. He strongly believed, along the Pythagorean tradition, that the universe should be described by mathematics. He also adopted the view, following Ockham's razor (Sect. **??**), that given various explanations of a phenomenon, the most succinct and economic one was more likely to be the correct one. Still any model must be tested again and again against experiment: no matter how beautifuland economical a theory is, should it fail to describe the data, it is useless except, perhaps, as a lesson.

4.2.1 Galilean relativity

Imagine a person inside a ship which is sailing on a perfectly smooth lake at constant speed. This passeneger is in the ship's windowless hull and, despite it being a fine day, is engaged in doing mechanical experiments (such as studying the behavior of pendula and the trajectories of falling bodies). A simple question one can ask of this researcher is whether she can determine that the ship is moving (with respect to the lake shore) *without going on deck or looking out a porthole*.

Since the ship is moving at constant speed and direction she will not *feel* the motion of the ship. This is the same situation as when flying on a plane: one cannot tell, without looking out one of the windows, that the plane is moving once it reaches cruising altitutde (at which point the plane is flying at constant speed and direction). Still one might wonder whether the experiments being done in the ship's hull will give some indication of the its motion. Based on his experiments Galileo concluded that this is in fact impossible: all mechanical experiments done inside a ship moving at constant speed in a constant direction would give precisely the same results as similar experiments done on shore.

The conclusion is that one observer in a house by the shore and another in the ship will not be able to determine that the ship is moving by comparing the results of experiments done inside the house and ship. In order to determine motion these observers must look at each other. It is important important to note that this is true *only* if the ship is sailing at constant speed and direction, should it speed up, slow down or turn the researcher inside *can* tell that the ship is moving. For example, if the ship turns you can see all things hanging from the roof (such as a lamp) tilting with respect to the floor

Generalizing these observations Galileo postulated his **relativity hypothesis**:

> any two observers moving at constant speed and direction with respect to one another will obtain the same results for all mechanical experiments

(it is understood that the apparatuses they use for these experiments move with them).

In pursuing these ideas Galileo used the scientific method (Sec. **??**): he derived consequences of this hypothesis and determined whether they agree with the predictions.

Velocity is *not* absolute

This idea has a very important consequence: *velocity is* not *absolute*. This means that velocity can only be measured in reference to some object(s), and that the result of this measurment changes if we decide to measure the velocity with respect to a diferent refernce point(s). Imagine an observer traveling inside a windowless spaceship moving away from the sun at constant velocity. Galileo asserted that there are no mechanical experiments that can be made inside the rocket that will tell the occupants that the rocket is moving . In fact, the question "are we moving" has no meaning unless we specify a reference point ("are we moving with respect to that star" *is meaningful*). This fact, formulated in the 1600's remains very true today and is one of the cornerstones of Einstein's theories of relativity.

> *Turbulence. (from Relativity and Its Roots,* by B. Hoffmann*).* Although this question will seem silly, consider it anyway: Why do the flight attendants on an airplane not serve meals when the air is turbulent but wait until the turbulence has passed?
> The reason is obvious. If you tried to drink a cup of coffee during a turbulent flight, you would probably spill it all over the place.
> The question may seem utterly inane. But even so, let us not be satisfied with only a partial answer. The question has a second part: Why is it all right for the flight attendants to serve meals when the turbulence has passed?
> Again the reason is obvious. When the plane is in smooth flight, we can eat and drink in it as easily as we could if it were at rest on the ground.
> Yes indeed! And *that* is a most remarkable fact of experience. Think of it.

A concept associated with these ideas is the one of a "frame of reference". We intuitively know that the position of a small body relative to a reference point is determined by three numbers. Indeed consider three long rods at 90^o from one another, the position of an object is uniquely determined by the distance along each of the corresponding three directions one must travel in order to get from the point where the rods join to the object (Fig. 4.1)

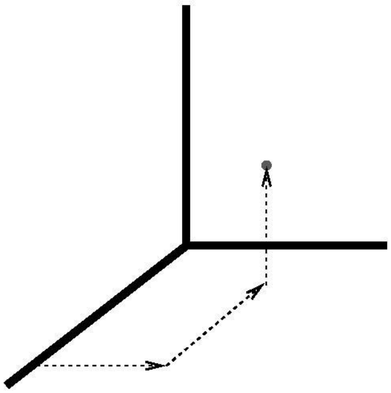

Figure 4.1: A frame of reference.

Thus anyone can determine positions and, if he/she carries clocks, motion of particles accurately by using these rods and good clocks. This set of rods and clock is called a *reference frame*. In short: a reference frame determines the where and when of anything with respect to a reference point.

A reference frame determines the where and when of anything with respect to a reference point.

A prediction of Galileo's principle of relativity is that free objects will move in straight lines at constant speed. A free object does not suffer form interactions from other bodies or agencies, so if it is at one time at rest in some reference frame, it will remain at rest forever in this frame. Now, imagine observing the body form another reference frame moving at constant speed and direction with respect to the first. In this second frame the free body is seen to move at constant speed and (opposite) direction. Still nothing has been done to the body itself, we are merely looking at it from another reference frame. So, in one frame the body is stationary, in another frame it moves at constant speed and direction. On the other hand if the body is influenced by something or other it will change its motion by speeding up, slowing down or turning. In this case either speed or direction are not constant as observed in ¡EM¿any¡/EM¿ reference frame. From these arguments Galileo concluded that free bodies are uniquely characterized by moving at constant speed (which might be zero) and direction.

An interesting sideline about Galilean relativity is the following. Up to that time the perennial question was, what kept a body moving? Galileo realized that this was the *wrong question*, since uniform motion in a straight line is not an absolute concept. The right question is, what keeps a body from moving uniformly in a straight line? The answer to that is "forces" (which are defined by these statements). This illustrates a big problem in physics, we have at our disposal all the answers (Nature is before us), but only when the right questions are asked the regularity of the answers before us becomes apparent. Einstein was able to ask a different set of questions and this lead to perhaps the most beautiful insights into the workings of Nature that have been obtained.

Galilean relativity predicts that free motion is in a straight line at constant speed. This important conclusion cannot be accepted without experimental evidence. Though everyday experience seems to contradict this conclusion (for example, if we kick a ball, it will eventually stop), Galileo realized that this is due to the fact that in such motions the objects are *not* left alone: they are affected by friction. He then performed a series of experiments in which he determined that frictionless motion would indeed be in a straight line at constant speed. Consider a ball rolling in a smooth bowl (Fig. 4.2).

Figure 4.2: Illustration of Galileo's experiments with friction

The ball rolls from it's release point to the opposite end and back to a certain place slightly below the initial point. As the surfaces of the bowl and ball are made smoother and smoother the ball returns to a point closer and closer to the initial one. In the limit of zero friction, he concluded, the ball would endlessly go back and forth in this bowl.

Following this reasoning and "abstracting away" frictional effects he concluded that

Free horizontal motion is constant in speed and direction.

Free horizontal motion is constant in speed and direction

This directly contradicts the Aristotelian philosophy which claimed that

- all objects on Earth, being imperfect, will naturally slow down,
- that in a vacuum infinite speeds would ensue,
- and that perfect celestial bodies must move in circles.

In fact objects on Earth slow down due to friction, an object at rest would stay at rest even if in vacuum, and celestial bodies, as anything else, move in a straight line at constant speed or remain at rest unless acted by forces.

4.2.2 Mechanics

Most of Galileo's investigations in physics had to do with the motion of bodies; these investigations lead him to the modern description of motion in terms of position and time. He realized that two important quantities that describe the motion of all bodies are velocity (which determines how position changes with time) and acceleration (which determines how velocity changes with time)

Velocity tells how position changes with time

Acceleration tells how velocity changes with time

Two important definitions:

Velocity: the rate of change of position, (how position changes with time).	*Acceleration*: the rate of change of velocity, (how velocity changes with time).

The motion of falling bodies

Galileo realized, even during his earliest studies (published in his book *On motion*) that the speed of a falling body is *independent* of its weight [1]. He argued as follows: suppose, as Aristotle did, that the manner in which a body falls does depend on it weight (or on some other quality, such as its "fiery" or "earthy" character), then, for example, a two pound rock should fall faster than a one pound rock. But if we take a two pound rock, split it in half and join the halves by a light string then one the one hand this contraption should fall as fast as a two pound rock, but on the other hand it should fall as fast as a one-pound rock (see Fig. 4.3). Since any object should have a definite speed as it falls, this argument shows that the Aristotle's assumption that the speed of falling bodies is determined by their weight is inconsistent; it is simply wrong. Two bodies released from a given height will reach the ground (in general) at different times not because they have different "earthliness" and "fiery" characteristics, but merely because they are affected by air friction differently. If the experiment is tried in vacuum *any* two objects when released from a given height, will reach the ground simultaneously (this was verified by the Apollo astronauts on the Moon using a feather and a wrench).

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[1] Galileo allegedly demonstrated his conclusions by dropping weights from the leaning tower of Pisa though this has been doubted by historians.

This result is peculiar to gravity, other forces do not beahve like this at all. For example, if you kick two objects (thus applying a force to them) the heavier one will move more slowly than the lighter one. In contrast, objects being affected by gravity (and starting with the same speed) will have the same speed at all times. This unique property of gravity was one of the motivations for Einstein's general theory of relativity (Chap. **??**).

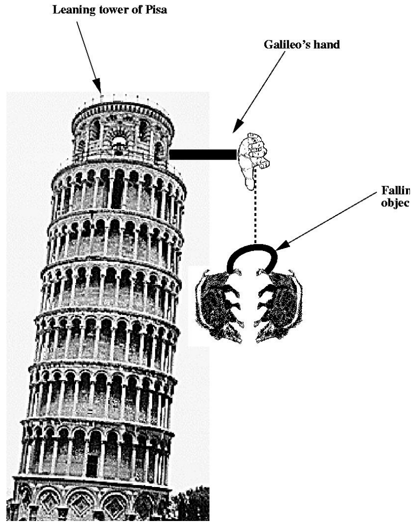

Figure 4.3: Illustration of Galileo's experiments with falling bodies.

Also in his investigations of falling bodies Galileo determined that the acceleration of these bodies is constant. He demonstrated that an object released from a height starts with zero velocity and increases its speed with time (before him it was thought that bodies when released acquire instantaneously a velocity which remained constant but was larger the heavier the object was). Experimenting with inclined planes, and measuring a ball's positions after equal time intervals Galileo discovered the mathematical expression of the law of falling bodies: the distance traveled increases as the *square* of the time.

The motion of projectiles

Galileo also considered the motion of projectiles. He showed that their motion can be decomposed in a motion along a vertical and horizontal directions. Thus if a ball is thrown horizontally (and air friction is ignored) it will move in the horizontal direction with constant speed; in the vertical direction it will experience the pull of gravity and will undergo free fall. The use of this can be illustrated by the following situation. Suppose a ball is let fall from a height h and is found to take t seconds to reach the ground. Now suppose that the ball is instead thrown horizontally with speed v, what distance will it cover? The answer is vt because the ball, even though it is moving horizontally, in the vertical direction is still freely falling: the two motions are completely independent! (see Fig. 4.4). This, of course, was of great use in warfare.

Motions along perpendicular directions are completely independent

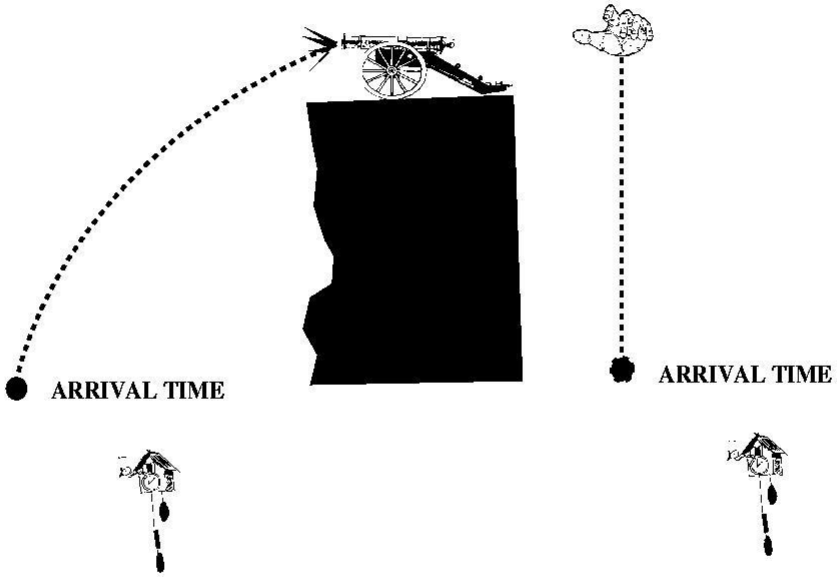

Figure 4.4: Horizontal and vertical motion are independent: the cannon shoots the ball *horizontally* at the same time the hand drops its ball; they both hit the ground at the same time.

As another experiment consider the “shoot the monkey” demonstration (Fig. 4.5). The setup is the following: a hunter wants to shoot a monkey who is hanging from a branch. As soon as he shoots the monkey lets go of the branch (thinking that the hunter aimed at the branch, he believes that the bullet will miss him). But the bullet, to the monkey’s surprise (and distress), does hit him! [2]

The reason is the following: if there were no forces the bullet would go in a straight line (as indicated by the dotted line in the figure) and the monkey would not fall. So the bullet would hit the monkey. Now, since

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[2] No real animals were hurt in this demonstration.

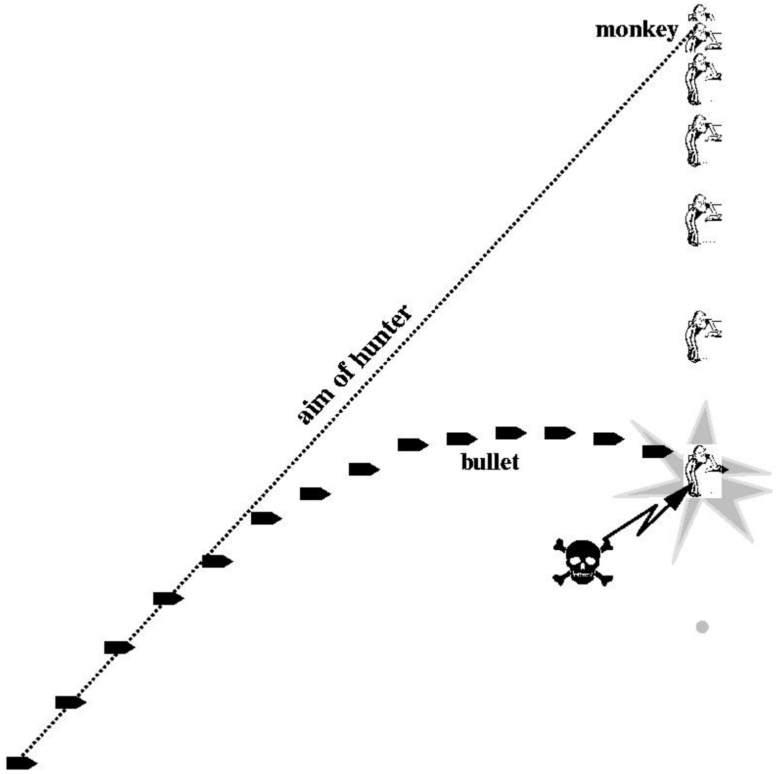

Figure 4.5: Shoot the "monkey": an illustration of motion in two dimensions.

we have a force acting on the system (gravity) the monkey will not stay at rest but will accelerate downward. But precisely the same force acts on the bullet in precisely the same way, hence the bullet will not go in a straight line but will follow the curve indicated in the figure. The deviation from their force-free motions (rest for the monkey, straight line for the bullet) are produced by a force which generates the same acceleration in both objects, hence these deviations are precisely matched in such a way that the bullet hits the monkey.

Now, given a force of *constant strength*, it will affect bodies in varying degrees; the more massive the object the smaller the effect: a blow from a hammer will send a small ball flying, the same blow will hardly affect a planet. On the other hand gravity produces the same *acceleration* on the monkey and the bullet; that is why the monkey is hit. Since the mass of the monkey is very different from that of the bullet we conclude that gravity's *force* is very different for each of them. The fact that the accelerations are independent of the mass but the force is not is actually a very profound fact: the whole of general relativity is based on it (Chap. **??**).

4.2.3 Astronomy

Throughout his life Galileo would provide some of the most compelling arguments in favor of the heliocentric model; though this brought him endless trouble in his lifetime, he was vindicated by all subsequent investigators. The beginnings of Galileo's astronomical studies were quite dramatic: in 1604 a "new star" (a supernova—an exploding star) was observed,. Galileo demonstrated that this object must lie beyond the Moon, contradicting the Aristotelian doctrine which claimed that the region beyond the Moon was perfect and unchanging. Yet here was a star that was not there before and would soon disappear!

A few years later he learned about the discovery of the telescope. He quickly realized its potential as a tool in astronomical research, and constructed several of them (Fig. 4.6), which he used to investigate the heavens.

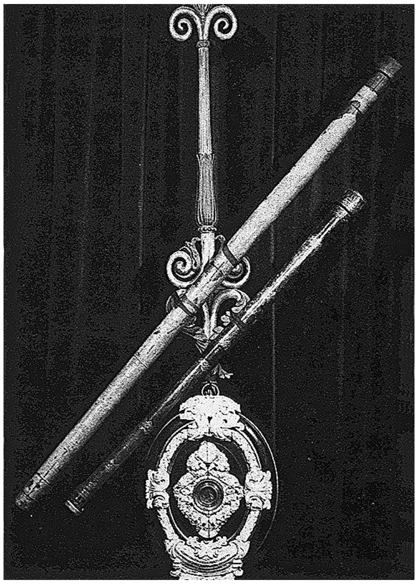

Figure 4.6: One of Galileo's telescopes

The first object which he studied with his telescope was the Moon of which he made many drawings (Fig. 4.7) some of which are quite accurate. He found that the surface of the Moon was heavily scarred, and identified some of the dark features he observed as shadows. The Moon was not exactly spherical and hardly perfect.

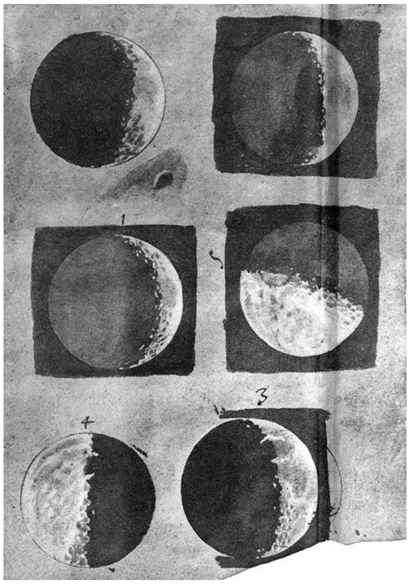

Figure 4.7: Galileo's drawings of the Moon.

Galileo was the first person to discover that Venus, like the Moon, shows periodic phases (Fig. 4.8). The simplest explanation is that this planet goes around the sun in accordance with the Copernican system. Galileo's astronomical observations were later verified by the Jesuit mathematicians of the Collegio Romano (although they did not necessarily agree with Galileo's interpretation!).

But the most dramatic of Galileo's astronomical discoveries was that of Jupiter's satellites (1610) [3]. He found that Jupiter was surrounded by a swarm of bodies that circled *it* and not Earth! These satellites, together with Jupiter, formed a mini-version of the Copernican model of the solar system with Jupiter taking the place of the Sun and it's satellites the places of the planets. All this was in blatant contradiction of the Aristotelian model; any remaining doubts which he might have had in his belief of the heliocentric model vanished.

Jupiter and its satellites formed a mini-version of the Copernican model of the solar system with Jupiter taking the place of the Sun and it's satellites the places of the planets

In 1613, in a book on sunspots, Galileo openly declared the Earth to circle the Sun. But by then the Church was getting worried about these

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[3] This landed him a permanent position as "Chief Mathematician of the University of Pisa and Philosopher and Mathematician to the Grand Duke of Tuscany"

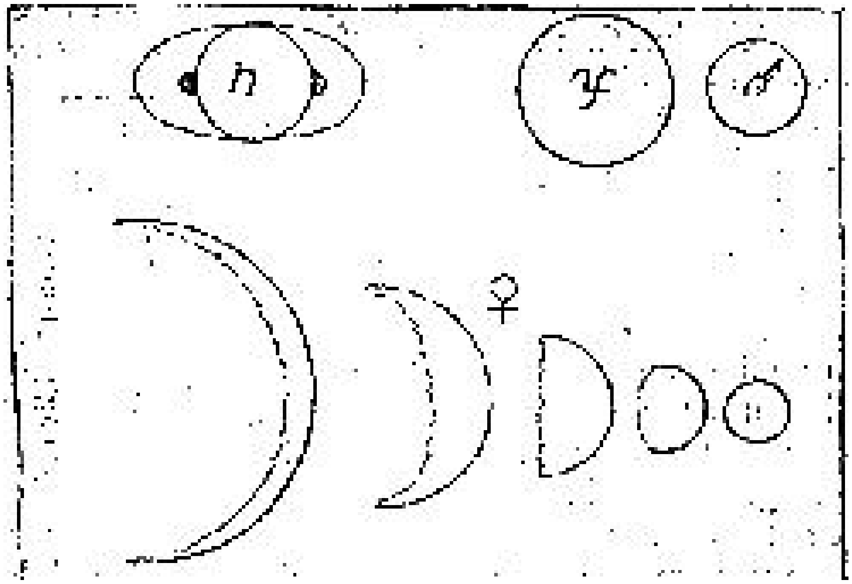


Figure 4.8: Galileo's drawings of the phases of Venus.

ideas: in 1616 Pope Pius V declared the Earth to be at rest and labeled the heliocentric model heretical, Copernicus' magnum opus was black-listed (where it remained until 1822!), and Galileo was called to Rome and told not to defend Copernicus' ideas.

In 1632 Galileo published his book on the Copernican and Ptolemaic systems *Dialogue Concerning the Two Chief Systems of the World* (in Italian so everyone could understand it). This was originally condoned by the Church, but the Pope Urban VIII had a change of heart and forbade the distribution of the book. Galileo was summoned to appear before the Roman Inquisition where, in a penitential garb and on one knee, he was made to swear on the Bible that he

> "...abjured, cursed, and detested the error and heresy that the Sun is fixed and the Earth moves"

and that he would no longer support this idea in any manner. He was put under house arrest and was made to recite the seven penitential psalms weekly for three years. This, of course, did not change the fact that the planets do move around the sun, but it embittered Galielo's last years.

4.2.4 Galileo and the Inquisition

Being one of the most renowned scientist of his time Galileo's opinions were scrutinized not only be his peers, but by also by Church officials and the public in general. This made Galileo the lightning-rod of many complaints against the Copernican doctrine (and also some against Galileo himself). He did not come out unscathed out of these encounters.

In 1611 Galileo came to the attention of the Inquisition for the first time for his Copernican views. Four years later a Dominican friar, Niccolo Lorini, who had earlier criticized Galileo's view in private conversations, files a written complaint with the Inquisition against Galileo's Copernican views. Galileo subsequently writes a long letter defending his views to Monsignor Piero Dini, a well connected official in the Vatican, he then writes his *Letter to the Grand Duchess Christina* arguing for freedom of inquiry and travels to Rome to defend his ideas

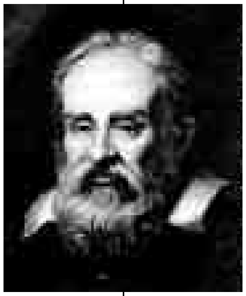
In 1616 a committee of consultants declares to the Inquisition that the propositions that the Sun is the center of the universe and that the Earth has an annual motion are absurd in philosophy, at least erroneous in theology, and formally a heresy. On orders of the Pope Paul V, Cardinal Bellarmine calls Galileo to his residence and administers a warning not to hold or defend the Copernican theory; Galileo is also forbidden to discuss the theory orally or in writing. Yet he is reassured by Pope Paul V and by Cardinal Bellarmine that he has not been on trial nor being condemned by the Inquisition.

In 1624 Galileo meets repeatedly with his (at that time) friend and patron Pope Urban VIII, he is allowed to write about the Copernican theory as long as he treated it as a mathematical hypothesis.

In 1625 a complaint against Galileo's publication *The Assayer* is lodged at the Inquisition by a person unknown. The complaint charges that the atomistic theory embraced in this book cannot be reconciled with the official church doctrine regarding the Eucharist, in which bread and wine are "transubstantiated" into Christ's flesh and blood. After an investigation by the Inquisition, Galileo is cleared.

In 1630 he completed his book *Dialogue Concerning the Two Chief World Systems* in which the Ptolemaic and Copernican models are discussed and compared and was cleared (conditionally) to publish it by the Vatican. The book was printed in 1632 but Pope Urban VIII, convinced by the arguments of various Church officials, stopped its distribution; the case is referred to the Inquisition and Galileo was summoned to Rome despite his infirmities.

Galileo Galilei (Feb. 15, 1564–1642). Born near Pisa, Italy, died near Florence, Italy. In 1581 he matriculates as a student of the Arts at the University of Pisa (his father's wish is that he study medicine) and he is first introduced to Euclid's Elements while studying in Florence under the court mathematician Ostilio Ricci. In 1585 he returns to Florence without a degree. He gives private lessons in mathematics until 1589; he begins his studies in physics. In 1588 he obtained a lectureship of mathematics at the Univ. of Pisa where he taught until 1592; he publishes *On motion*. In 1592 Galileo obtains the chair of mathematics at the University of Padua in the Venetian Republic where he remains until 1610.



In 1599 he enters a relationship with Marina Gamba with whom he had three children, two daughters and one son. The daughters were placed in a convent as Galileo could not provide adequate dowries; he eventually managed to have his son legitimated. In 1613 Marina Gamba married Giovanni Bartoluzzi, it appears that Galileo kept cordial relations with Gamba and Bartoluzzi.

In 1609, he observes (using telescopes of his construction) the Moon, and discovers 4 satellites around Jupiter. In this year he was also appointed (for life) "Chief Mathematician of the University of Pisa and Philosopher and Mathematician to the Grand Duke of Tuscany". In 1611 he is admitted to the Lycean Academy and came to the attention of the Inquisition for the first time . In 1615 he is denounced to the the Inquisition, he defends himself in the *Letter to the Grand Duchess Christina*. In 1616 the Copernican doctrine is declared heretical, Galileo is warned against supporting this theory either orally, but he is allowed to write about it as a mathematical hypothesis. In 1621 Galileo is elected Consul of the Accademia Fiorentino. In 1625 a complaint to the Inquisition against Galileo's publication *The Assayer* is lodged by a person unknown; the complaint charges that the atomistic theory embraced in this book is heretical; Galileo is cleared.

In 1630 completes his book *Dialogue Concerning the Two Chief World Systems* contrasting the Ptolemaic and Copernican models. The book was printed in 1632 but the Pope Urban VIII stopped its distribution; the case is referred to the Inquisition and Galileo was summoned to Rome despite his physical infirmities. A year later Galileo is formally interrogated by the Inquisition. He recants of his support of the Copernican model and is ordered held under house arrest where he would remain until his death; also in 1633 he begins writing his *Discourse on Two New Sciences*. His health deteriorates steadily, in 1634 he suffers a painful hernia, by 1638 he is totally blind. Galileo dies in Arcetri on 8 January 1642.

Galileo also invented several objects of great practical interest such as an hydrostatic balance (1608), a horse-driven water pump (1593), a geometric and military compass (1597), various telescopes (1609) and a thermometer (1606). In 1641 he conceives of the application of the pendulum to clocks.

In 1633 Galileo was formally interrogated for 18 days and on April 30 Galileo confesses that he may have made the Copernican case in the Dialogue too strong and offers to refute it in his next book. Unmoved, the Pope decides that Galileo should be imprisoned indefinitely. Soon after, with a formal threat of torture, Galileo is examined by the Inquisition and sentenced to prison and religious penances, the sentence is signed by 6 of the 10 inquisitors. In a formal ceremony at a the church of Santa Maria Sofia Minerva, Galileo abjures his errors. He is then put in house arrest in Sienna. After these tribulations he begins writing his *Discourse on Two New Sciences*.

Galileo remained under house arrest, despite many medical problems and a deteriorating state of health, until his death in 1642. The Church finally accepted that Galileo might be right in 1983.

4.3 Isaac Newton

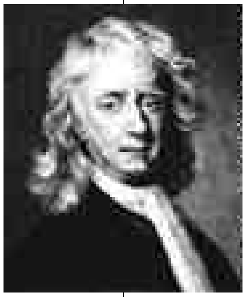
On Christmas day 1642, in the manor house of Woolsthorpe, a weak child was born and christened Isaac. He was to become the most influential scientist of the next 250 years. Isaac Newton discovered the laws that explained all phenomena known at the time, form the motion of the stars to the behavior of dust particles. It was his extremely successful model that lead people to believe that humanity was on the verge of understanding the whole of Nature.

Newton's life can be divided into three quite distinct periods. The first is his boyhood days from 1642 up 1665 when the Plague forced him to leave Cambridge. The second period from 1665 to 1687 was the highly productive period in which he became Lucasian professor at Cambridge. The third period (nearly as long as the other two combined) saw Newton as a highly paid government official in London with little further interest in science and mathematics.

I will talk about Newton quite a bit because his view of the world together with the mathematical formalism he developed lasted for 200 years: the first experimental results incompatible with it were obtained at the end of the XIX-th century and the whole structure was shown not to be fundamentally correct by 1925. One nonetheless should be aware of the fact that, while not perfectly correct, the results using the Newtonian are exceedingly accurate in all every-day applications. Newton's theory is not "wrong" it's just that it has a limited range of validity.

Isaac Newton (1643–1727). Born in the manor house of Woolsthorpe, near Grantham in Lincolnshire on Christmas Day 1642. Newton came from a family of farmers; his father died before he was born. His mother remarried, moved to a nearby village, and left him in the care of his grandmother. Upon the death of his stepfather in 1656, Newton's mother removed him from grammar school in Grantham where he had shown little promise in academic work. His school reports described him as 'idle' and 'inattentive'. Legend has it that one day the student just ahead of him in class kicked him in the stomach, Newton won the fight and he also decided to get ahead of this student in class ranking. He succeeded admirably. An uncle decided that he should be prepared for the university, and he entered his uncle's old College, Trinity College, Cambridge, in June 1661.

Instruction at Cambridge was dominated by the philosophy of Aristotle but some freedom of study was allowed in the third year of study. Newton's aim at Cambridge was a law degree, yet he also studied the philosophy and analytical geometry of Descartes, Boyle's works, and the mechanics of the heliocentric astronomy of Galileo.



His scientific genius flourished suddenly when the "Black Death" plague closed the University in the summer of 1665 and he had to return to Lincolnshire. There, in a period of less than two years, while Newton was still under 25 years old, he began revolutionary advances in optics, physics, and astronomy. In mathematics he laid the foundation for differential and integral calculus several years before its independent discovery by Leibniz. (this work, *De Methodis Serierum et Fluxionum*, was written in 1671 but appeared only 60 years later).

Impressed with Newton's abilities, Barrow resigned the Lucasian chair in 1669 recommending that Newton (still only 27 years old) be appointed in his place. Newton's first work as Lucasian Professor was on optics. Newton was elected a fellow of the Royal Society in 1672 after donating a reflecting telescope. In that year he published his first scientific paper on light and color in the Philosophical Transactions of the Royal Society.

Newton's relations with the influential scientist Robert Hooke deteriorated and Newton turned away from the Royal Society and mainstream science; he delayed the publication of a full account of his optical researches until after Hooke's death in 1703: Newton's *Opticks* appeared in 1704.

Newton's greatest achievement was his work in physics and celestial mechanics, which culminated in the theory of universal gravitation. His results are summarized in his treatise of physics *Philosophiae Naturalis Principia Mathematica* which appeared in 1687.

After suffering a nervous breakdown in 1693, Newton retired from research to take up a government position in London becoming Warden of the Royal Mint (1696) and Master (1699). In 1703 he was elected president of the Royal Society and was re-elected each year until his death. He was knighted in 1708 by Queen Anne, the first scientist to be so honored for his work. Newton died in 1727; his tomb in Westminster Abbey is inscribed with these words: " Mortals! Rejoice at so great an ornament to the human race!"

4.3.1 Mechanics.

During the years of the Plague Newton constructed what was to become an remarkably successful model of Nature. In it he proposed three laws that describe the motion of all material bodies (at least for all phenomena within reach at the time). These were not mere descriptions but actual calculational tools, and the enormous accuracy in the predictions achieved by this theory resulted in its universal acceptance that lasted more than two centuries...until Einstein came along.

After returning to Cambridge, Newton lost interest in mechanics until 1684. In this year Halley, tired of Hooke's boasting, asked Newton whether he could prove Hooke's conjecture that planets moved in ellipses because the sun attracted them with a force decreasing as the square of the distance. Newton told him that he had indeed solved this problem five years earlier, but had now mislaid the proof. At Halley's urging Newton reproduced the proofs and expanded them into a paper on the laws of motion and problems of orbital mechanics. Halley then persuaded Newton to write a full treatment of his new physics and its application to astronomy. Over a year later (in 1687) Newton published the *Philosophiae Naturalis Principia Mathematica* or the *Principia* as it is commonly known. It is one of the greatest scientific books ever written.

Newton laid in his Principia three laws which describe the motion of bodies. These laws have an immense range of applicability, failing only at very small distances (of 10^{-8}cm or less), for very strong gravitational fields (about 10^8 stronger than the Sun's), or for very large speeds (near 10^8 m/s).

The first of Newton's laws addresses the motion of free bodies. The second law states quantitatively how a motion differs form free motion. The third law states the effect experienced by a body when exerting a force on another object.

- *1st law.* Every body continues its state of rest or uniform motion in a straight line unless it is compelled to change this state by forces acting on it.

Free bodies move in straight lines or remain at rest

- *2nd law.* The effect of a force F on the motion of a body of mass m is given by the relation

  $$F = ma$$

  where a is the acceleration: a body in the presence of a force F attains an acceleration equal to F/m.

Force=mass×acceleration

- *3rd law.* Every body exerting a force on another, experiences a force exerted by the second body equal in magnitude and in opposite direction.

Action=reaction

These three laws constitute Newton's basic hypothesis He asserted that they are valid in all circumstances and to all bodies, in particular for heavenly bodies as well as for earth objects; this marks the final passing of Aristotelian physics. All experimental evidence of the time (and for the next two centuries) was to support these hypothesis, Newton's theory became *the* theory of Nature.

I will now discuss some of the features of these laws.

1st Law and Newtonian space and time.

One of the most important consequences of the First Law is that it *defines* what we mean by an inertial frame of reference.

> An inertial reference frame is a reference frame where isolated bodies are seen to move in straight lines at constant velocity.

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An observer at rest with respect to an inertial frame of reference is called an *inertial observer*. The laws of physics devised by Newton take a particularly simple form when expressed in terms of quantities measured by an inertial observer (such as positions, velocities, etc.). For example, an inertial observer will find that a body on which no forces act moves in a straight line at constant speed or is at rest.

All motion occurs in space and is measured by time. In Newton's model both space and time are unaffected by the presence or absence of objects. That is *space and time are absolute*, an arena where the play of Nature unfolds. In Newton's words,

Newton assumed that space and time are absolute

> *Absolute space in its own nature, without relation to anything external, remains always similar and immovable.*

> *...absolute and mathematical time, of itself, and from its own nature, flows equally without relation to anything external, and by another name is called duration.*

Space and time were taken to be featureless objects which served as a universal and preferred reference frame (see Fig. 4.9 for an illustration). A consequence of this is that a given distance will be agreed upon by any two observers at rest with respect to each other or in uniform relative motion, for, after all, they are just measuring the separation between two immovable points in eternal space. In the same way a time interval will be agreed upon by *any* two observers for they are just marking two notches on eternal time.

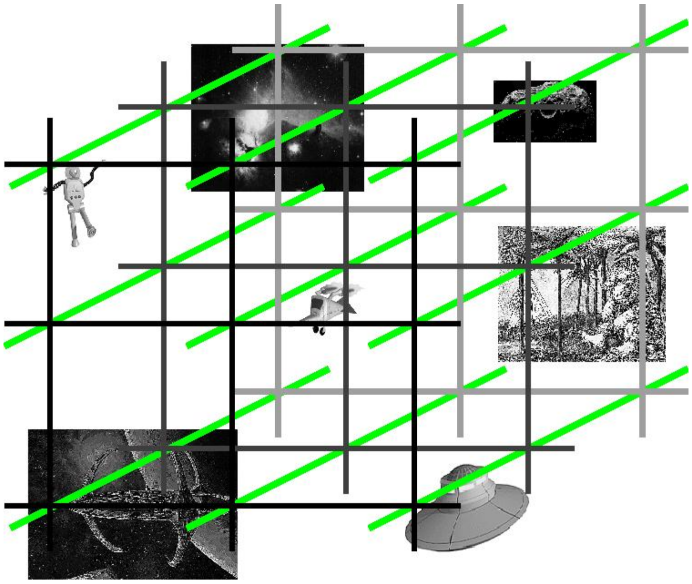

Figure 4.9: Illustration of Newton's concept of space. The grids represent space which are unaffected by the presence and properties of the objects in it.

Newton's assumptions about space and time are the foundation of his theory of Nature and were accepted due to the enormous successes of the predictions. Eventually, however, experimental results appeared which disagreed with the predictions derived from Newton's theory. These problems were traced to the fact that these basic assumptions are not accurate descriptions of space and time (though they do represent a very good approximation): space and time are not absolute (Chaps. **??**, **??**) [4]. The realization that Newton's theory required revisions came to a head at the beginning of the XXth century. In the two decades from 1905 to 1925 a completely new framework was constructed and has now replaced Newton's ideas. These theories comprise the special and general theories of relativity and quantum mechanics.

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[4] $F = ma$ is also not universally valid but deviations from this expression occur only at very small distances and can be understood in the framework of Quantum Mechanics.

Do we know that the current theories of space and time are the truth? The answer is no: we do know that the current theories explain all the data (including the one explained by Newton and more), but we cannot determine whether they represent the ultimate theories of Nature. In fact, we expect them not to be the last word as there are many unexplained questions; for example, why should the proton be precisely 1836.153 times heavier than the electron? Why should space have 3 and not 25 dimensions? etc.

But in the 17th century there was no inkling of these problems and very few scientist questioned Newton's hypothesis. In particular Newton constructed his mechanics to comply with Galilean relativity: an observer in uniform motion with respect to another cannot, without looking outside his laboratory, determine whether he is at rest or not. And even if he looks outside, he cannot decide whether he is in motion or the other observer is. In fact for two inertial observers moving relative to each other the question, "which of us is moving?" is un-answerable and meaningless. The only thing to be said is that they have a certain relative velocity.

2nd Law

The second law is of great practical use. One can use experiments to determine the manner in which the force depends on the position and velocity of the bodies and then use calculus (which was also invented by Newton) to determine the motion of the bodies by obtaining the position as a function of time using the known form of F and the equation $F = ma$. Note that in this equation m measures how strongly a body responds to a given force (the larger m is the less it will be accelerated); m measures the inertia of the body.

Suppose we choose a test body of mass, say, 1gm. By measuring its motion one can obtain its acceleration and, using $F = ma$, determine the force. Once F is known the motion of *any* body is predicted: by measuring the falling an apple you can predict the motion of the Moon.

3rd Law

The third law is, at first sight, almost unbelievable: if I kick a ball, the ball kicks me back? But in fact it *is* so: suppose I push a friend while we are both standing on ice (to minimize friction), then he/she will move in the direction of the push, but I will move backward! What happens when I kick a ball is that the push backward is countered by the friction between my other leg and the ground, and because of this no motion backward ensues.

It is interesting to do the kick-the-ball experiment on ice, you should try it.

4.3.2 Optics

Newton's first work as Lucasian Professor was on optics. Every scientist since Aristotle had believed light to be a simple entity, but Newton, through his experience when building telescopes, believed otherwise: it is often found that the observed images have colored rings around them (in fact, he devised the reflecting telescope, Fig. 4.10, to minimize this effect). His crucial experiment showing that white light is composite consisted in taking beam of white light and passing it through a prism; the result is a wide beam displaying a spectrum of colors. If this wide beam is made to pass through a second prism, the output is again a narrow beam of white light. If, however, only one color is allowed to pass (using a screen), the beam after the second prism has this one color again. Newton concluded that white light is really a mixture of many different types of colored rays, and that these colored rays are not composed of more basic entities (see Fig. 4.11).

Figure 4.10: Newton's first reflective telescope.

Concerning the nature of light. Newton believed that it consists of a stream of small particles (or corpuscules) rather than waves. Perhaps because of Newton's already high reputation this "corpuscular" theory was accepted until the wave theory of light was revived in the 19th C.

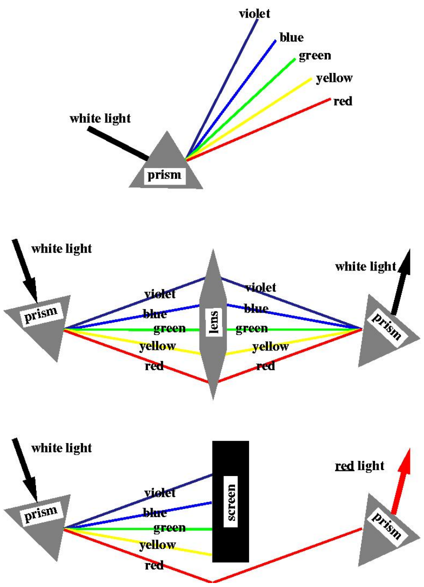

Figure 4.11: Diagram of Newton's experiments on the composition of white light.

4.3.3 Gravitation.

One of Newton's greatest achievements was on the field of celestial mechanics where he produced the first synthesis in the theories describing Nature: he realized that the same force that makes things fall, gravity, is responsible for the motion of the Moon around the Earth and the planets around the Sun.

He reasoned (more or less) as follows. Suppose I let an apple fall form a very high tower, it will take, say, t seconds to reach the ground. Now suppose I throw it very hard, then again it will take t seconds to reach the ground *provided* I assume the Earth is flat. But the Earth *isn't* flat and has curved from beneath the apple! Hence the apple will take longer to hit the ground. By throwing the apple with increasing force one reaches a point where the apple never hits the ground as the distance it falls equals the distance the earth has curved under it: the apple is in orbit! (see Fig. 4.12)

With this thought experiment Newton convincingly argued that an apple can behave in the same way as the Moon, and, because of this it is the very same force, gravity, which makes the apple fall and the Moon orbit the Earth. This is consistent with the hypothesis that gravitation is universal. In a way it represents the unification a several physical effects which appear

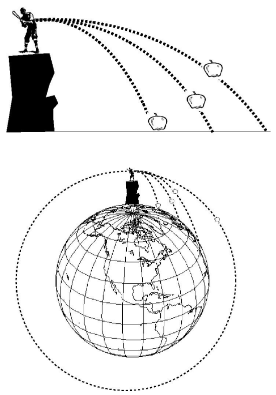

Figure 4.12: Newton's explanation of the equivalence between the force making apples fall and the one responsible for the Moon orbiting the Earth.

unrelated at first sight: the falling of apples and the orbiting of planets.

Having realized this he then used the results of Kepler and showed that if the planets and the sun are assumed to be point-like, the gravitational force drops as the inverse distance squared: the gravitational force between two bodies of masses m and M separated by a distance r is attractive and directed along the line joining the bodies, its value is

$$F_{\text{grav}} = \frac{mMG}{r^2}$$

where G is a universal constant, in words,

> *all matter attracts all other matter with a force proportional to the product of their masses and inversely proportional to the square of the distance between them.*

Having discovered this Newton was able to explain a wide range of previously unrelated phenomena: the eccentric orbits of comets, the tides and their variations, the precession of the Earth's axis, and motion of the Moon as perturbed by the gravity of the Sun. It also predicts the position of the planets for thousands of years so that the occurrence of eclipses can be foretold with exquisite accuracy, Moon landings can be planned without uncertainties, etc.

All matter attracts all other matter with a force proportional to the product of their masses and inversely proportional to the square of the distance between them

Consider now the application of the second law to the case of the gravitational force.

$$\frac{mMG}{r^2} = F_{\text{grav}} = ma$$

so that the factors of m cancel (!) This implies that the motion of a body generated by the gravitational force is *independent* of the mass of the body (!!), (just as Galileo had observed). This unique feature results from $F_{\rm grav}$ being precisely proportional to m. So m is seen made to play two roles:

- On the one hand m in $F = ma$ is a measure of how strongly is a body accelerated by a given force: it is a measure of the body's inertia. In this role m is called the *inertial mass.*
- On the other hand m in $F_{\rm grav}$ is a measure of how strongly is a body affected by the force of gravity and also how strong a gravitational force is generated by m; in this role it is called the *gravitational mass.*

These two quantities refer to different properties of a body and need not be equal. Extremely precise measurements, however, indicate that they *are* equal (at least to one part in ten parts per trillion). Newton just stated that this was the way of the world and kept going. Einstein, in contrast, noted this as a very important fact of nature, which he used to give birth to his General Theory of Relativity (Chap. **??**).

Concerning the nature of gravitation. there is another interesting feature of $F_{\rm grav}$: it is time independent. this implies that if a body moves, this change is perceived instantaneously by all the bodies throughout the universe. Leibnitz (among others) criticized Newton's hypothesis along these lines, and was disregarded. But this only due to the enormous success of Newtoninan gravity in making predictions of the motions of the bodies in the solar system. In fact we will see that this is not correct, and that the effect spreads out from the body at a finite speed (Chap. **??**).

To give an idea of the trust and excellent successes of Newtonian gravity consider the story of the discovery of Neptune. In 1843 a young astronomer at Cambridge, J.C. Adams discovered an anomaly in the orbit of Uranus and by the end of 1845 had concluded that this was due, not to a failure of Newton's law of gravity, but to the presence of a new planet. Adams submitted his results to G. Airy, his boss, who was unconvinced and dropped the matter. Meanwhile U. Leverrier in France had done a similar set of calculations independently, he published in 1846. This spurred Airy into action, but the Cambridge Observatory lacked an up to date chart of the region of the sky were the new planet was supposed to have resided at the time. During that time Leverrier wrote to J.G. Galle at the Berlin Observatory who promptly located the new planet. After much discussion this planet was called Neptune.