Due Tuesday November 14

19. (10 points) A charged particle travels a curved path in a magnetic field. It is, therefore, being accelerated and, classically, emits electromagnetic radiation. This radiation is observed in particle accelerators and is known as synchrotron radiation. The peak of this radiation occurs at a frequency corresponding to a photon energy of

\[ E \text{ (in GeV)} = \frac{\hbar c \gamma^3}{2R} \text{ (in m)} = 0.74 \times 10^{-6} \frac{E}{R} \text{ (in m)} \]

for electrons, and the radiation per turn is given by

\[ 4\pi r_e E^4/(3m^3R) = 4\pi e^2 \beta^3 \gamma^4/3R \text{ (in m)} \]

\[ = 8.85 \times 10^{-5} \frac{E}{R} \text{ (in m)} \]

for high-energy electrons, where \( r_e \) is the classical radius of the electron: \( r_e = e^2/m_e c^2 \). Find the energy of photons at the peak of the synchrotron radiation and the average energy/turn lost for the Fermilab Tevatron collider and for LEP. For the Tevatron collider, the particles are 1 TeV protons, and the circumference of the collider is 6.28 km. For LEP, the particles are 45.5 GeV electrons, and the circumference of the collider is 26.7 km. This radiation is the principal limitation on the size of circular electron machines.

20. (10 points) Does a rotation \( R \) commute with the parity operation \( P \)? Show why or why not.

21. (10 points) Show that the four successive infinitesimal rotations (\( \epsilon \) about the \( x \)-axis, followed by \( \eta \) about the \( y \)-axis, then \( -\epsilon \) about the \( x \)-axis, and finally \( -\eta \) about the \( y \)-axis) are equivalent to a second order rotation by \( (-\epsilon \eta) \) about the \( z \)-axis. Hence, show that the generators satisfy \([L_x, L_y] = iL_z\).

22. (10 points) The \( \eta \) meson is a neutral meson of mass 547.5 MeV that decays 39% of the time into two photons and most of the rest of the time into three pions (32% \( 3\pi^0 \), 24% \( \pi^+ \pi^- \pi^0 \)). The decay angular distribution indicates that the pions are in an \( S \) state: all pairs of pions have \( l = 0 \). There are no charged partners of the \( \eta \). Use some portion of the preceding information to find the spin and parity of the \( \eta \).

23. (10 points) Ignoring small phase space differences, find the branching ratios expected for \( \Sigma^+ \) decay into \( p\pi^0 \) and \( n\pi^+ \). Assume first that the final state is pure \( I = 3/2 \), and then redo the calculation for a final state that is pure \( I = 1/2 \). Experimentally, the ratio between the decay rates for \( p\pi^0 \) and \( n\pi^+ \) and is almost one.